SMARANDACHE HYPER K-ALGEBRAS

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ABSTRACT. We introduce the notion of an extension of hyper K-algebra and Smarandache hyper $(\cap, \in)$-ideal in hyper K-algebra, and investigate its properties.

1. INTRODUCTION

Generally, in any human field, a *Smarandache Structure* on a set A means a weak structure **W** on A such that there exists a proper subset B of A which is embedded with a strong structure **S**. In [10], W. B. Vasantha Kandasamy studied the concept of Smarandache groupoids, subgroupoids, ideal of groupoids, semi-normal subgroupoids, Smarandache Bol groupoids and strong Bol groupoids and obtained many interesting results about them. Smarandache semigroups are very important for the study of congruences, and it was studied by R. Padilla [9].

In this paper, we introduce the notion of an extension of hyper K-algebra and Smarandache hyper $K(\cap, \in)$-ideal in hyper K-algebra, and investigate its properties.

2. PRELIMINARIES

We include some elementary aspects of hyper K-algebras that are necessary for this paper, and for more details we refer to [1] and [11]. Let H be a non-empty set endowed with a hyper operation "$\circ$", that is, $\circ$ is a function from $H \times H$ to $\mathcal{P}^*(H) = \mathcal{P}(H) \setminus \{\emptyset\}$. For two subsets A and B of H, denote by $A \circ B$ the set $\bigcup_{a\in A, b\in B} a \circ b$.

By a *hyper BCK-algebra* we mean a non-empty set H endowed with a hyperoperation "$\circ$" and a constant 0 satisfying the following axioms:

(HK1) $(x \circ z) \circ (y \circ z) \ll x \circ y$,
(HK2) $(x \circ y) \circ z = (x \circ z) \circ y$,
(HK3) $x \circ H \ll \{x\}$,
(HK4) $x \ll y$ and $y \ll x$ imply $x = y$,

for all $x, y, z \in H$, where $x \ll y$ is defined by $0 \in x \circ y$ and for every $A, B \subseteq H$, $A \ll B$ is defined by $\forall a \in A$, $\exists b \in B$ such that $a \ll b$.

By a *hyper I-algebra* we mean a non-empty set H endowed with a hyper operation "$\circ$" and a constant 0 satisfying the following axioms:

(H1) $(x \circ z) \circ (y \circ z) < x \circ y$,
(H2) $(x \circ y) \circ z = (x \circ z) \circ y$,
(H3) $x < x$,
(H4) $x < y$ and $y < x$ imply $x = y$

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for all $x, y, z \in H$, where $x < y$ is defined by $0 \in x \circ y$ and for every $A, B \subseteq H$, $A < B$ is defined by $\exists a \in A$ and $\exists b \in B$ such that $a < b$. If a hyper I-algebra $(H, \circ, 0)$ satisfies an additional condition:

(H5) $0 < x$ for all $x \in H$,

then $(H, \circ, 0)$ is called a *hyper K-algebra* (see [1]).

Every hyper BCK-algebra is a hyper K-algebra. We know that there exists a proper hyper K-algebra, that is, there exists a hyper K-algebra which is not a hyper BCK-algebra (See [1, Theorem 3.5]).

In a hyper I-algebra H, the following hold (see [1, Proposition 3.4]):

(a1) $(A \circ B) \circ C = (A \circ C) \circ B$,
(a2) $A \circ B < C \Leftrightarrow A \circ C < B$,
(a3) $A \subseteq B$ implies $A < B$

for all non-empty subsets A, B and C of H.

In a hyper K-algebra H, the following holds (see [1, Proposition 3.6]):

(a4) $x \in x \circ 0$ for all $x \in H$.

Definition 2.1. ([1]) Let $(H, \circ, 0)$ be a hyper K-algebra and let S be a subset of H containing 0. If S is a hyper K-algebra with respect to the hyperoperation "$\circ$" on H, we say that S is a *hyper K-subalgebra* of H.

Note that if S be a non-empty subset of a hyper K-algebra $(H, \circ, 0)$, then S is a hyper K-subalgebra of H if and only if $x \circ y \subseteq S$ for all $x, y \in S$ (See [3, Theorem 4.12]).

Definition 2.2. ([3, Theorem 3.4]) A *Smarandache hyper K-algebra* is defined to be a hyper K-algebra $(H, \circ, 0)$ in which there exists a proper subset Ω of H such that $(\Omega, \circ, 0)$ is a non-trivial hyper BCK-algebra.

Example 2.3. ([3, Example 3.5]) Let $H = \{0, a, b, c\}$ and define an hyper operation "$\circ$" on H by the following Cayley table:

$\circ$	0	a	b	c
0	$\{0\}$	$\{0\}$	$\{0\}$	$\{0\}$
a	$\{a\}$	$\{0\}$	$\{0\}$	$\{0\}$
b	$\{b\}$	$\{a\}$	$\{0, a\}$	$\{0, a\}$
c	$\{c\}$	$\{a, b, c\}$	$\{a, b, c\}$	$\{0, b, c\}$

Table a3

Then $(H, \circ, 0)$ is a Smarandache hyper K-algebra because $(\Omega = \{0, a, b\}, \circ, 0)$ is a hyper BCK-algebra.

Example 2.4. ([3, Example 3.6]) Let $H = \{0, a, b\}$ and define an hyper operation "$\circ$" on H by the following Cayley table:

$\circ$	0	a	b
0	$\{0\}$	$\{0\}$	$\{0\}$
a	$\{a, b\}$	$\{0, a, b\}$	$\{0, a\}$
b	$\{b\}$	$\{a, b\}$	$\{0, a, b\}$

Table a4

Then $(H, \circ, 0)$ is not a Smarandache hyper K-algebra since $(\Omega_1 = \{0, a\}, \circ, 0)$ and $(\Omega_2 = \{0, b\}, \circ, 0)$ are not hyper BCK-algebras.

Definition 2.5. ([3, Definition 3.7]) Let H be a Smarandache hyper hyper K-algebra and Ω be a non-trivial hyper BCK-algebra which is properly contained in H. Then a non-empty subset I of H is called a *Smarandache hyper* $(\ll, \in)$-*ideal* of H related to Ω (or briefly, Ω-*Smarandache hyper* $(\ll, \in)$-*ideal* of H) if it satisfies:

(c1) $0 \in I$,
(c2) $(\forall x \in \Omega)\ (\forall y \in I)\ (x \circ y \ll I \Rightarrow x \in I)$.

If I is a Smarandache hyper $(\ll, \in)$-ideal of H related to every hyper BCK-algebra contained in H, we simply say that I is a *Smarandache hyper* $(\ll, \in)$-*ideal* of H.

Definition 2.6. ([3, Definition 3.14]) Let H be a Smarandache hyper hyper K-algebra and Ω be a non-trivial hyper BCK-algebra which is properly contained in H. Then a non-empty subset I of H is called a *Smarandache hyper* $(\subseteq, \in)$-*ideal* of H related to Ω (or briefly, Ω-*Smarandache hyper* $(\subseteq, \in)$-*ideal* of H) if it satisfies:

(c1) $0 \in I$,
(cw) $(\forall x \in \Omega)\ (\forall y \in I)\ (x \circ y \subseteq I \Rightarrow x \in I)$.

If I is a Smarandache hyper $(\subseteq, \in)$-ideal of H related to every hyper BCK-algebra contained in H, we simply say that I is a *Smarandache hyper* $(\subseteq, \in)$-*ideal* of H.

3. Main results

Proposition 3.1. *Let $(H, \circ, 0)$ be a hyper K-algebra with $|H| \geq 3$. Then the following statements hold:*

(i) *If there exists a hyper K-subalgebra S of H such that $1 < |S| < |H|$ and $|x \circ y| = 1$ for all $x, y \in S$, then H is a Smarandache hyper K-algebra.*
(ii) *If there exists $x \in H$ such that $x \circ x \subseteq \{0, x\}$, then H is a Smarandache hyper K-algebra.*

Proof. (i) Let S be a hyper K-subalgebra of H such that $2 \leq |S| < |H|$ and $|x \circ y| = 1$ for all $x, y \in S$. Then it can be easily verified that $(S, \circ, 0)$ is a hyper BCK-algebra. Therefore H is a Smarandache hyper K-algebra.

(ii) Let $x \in H$ be such that $x \circ x \subseteq \{0, x\}$. Note that $(\{0, x\}, \circ, 0)$ is a hyper BCK-algebra, and so H is a Smarandache hyper K-algebra. □

Example 3.2. The condition $|x \circ y| = 1$ for all $x, y \in S$ in the Proposition 3.1(i) is necessary. To show this, we consider $H = \{0, a, b\}$ in Example 2.4. Then $(S = \{0, a\}, \circ, 0)$ is a hyper K-algebra, but $(H, \circ, 0)$ is not a Smarandache hyper K-algebra.

Definition 3.3. Let $(H, \circ_H, 0)$ be a hyper K-algebra. By an *extension* of H we mean a hyper K-algebra $(L, \circ_L, 0)$ such that

(i) $H \subset L$,
(ii) $(\forall x, y \in H)(x \circ_H y = x \circ_L y)$.

Example 3.4. ([1, Theorem 3.7]) Let $(H_1, \circ_1, 0)$ and $(H_2, \circ_2, 0)$ be hyper K-algebras (resp. hyper BCK-algebras) such that $H_1 \cap H_2 = \{0\}$ and $H = H_1 \cup H_2$. Then $(H, \circ, 0)$ is a hyper K-algebra (resp. hyper BCK-algebra), where the hyperoperation "$\circ$" on H is defined as follows:

$$x \circ y := \begin{cases} x \circ_1 y & \text{if } x, y \in H_1, \\ x \circ_2 y & \text{if } x, y \in H_2, \\ \{x\} & \textit{oterwise} \end{cases}$$

for all $x, y \in H$.

We use the notation $H_1 \oplus H_2$ for the union of two hyper K-algebras (resp. hyper BCK-algebra) H_1 and H_2.

Theorem 3.5. *If H is a Smarandache hyper K-algebra, then every extension of H is also a Smarandache hyper K-algebra.*

Proof. Straightforward. □

The following example show that there exists a non-Smarandache hyper K-algebra H such that an extension L of H is a Smarandache hyper K-algebra.

Example 3.6. Let $(H = \{0, x\}, \circ_1, 0)$ be a hyper BCK-algebra and let $(K = \{0, y\}, \circ_2, 0)$ be a hyper K-algebra with the following Cayley tables:

$\circ_1$	0	x
0	$\{0\}$	$\{0\}$
x	$\{x\}$	$\{0, x\}$

$\circ_2$	0	y
0	$\{0\}$	$\{0\}$
y	$\{0, y\}$	$\{0\}$

Then $(L = H \oplus K, \circ, 0)$ is a Smarandache hyper K-algebra and it is an extension of H. But H is not a Smarandache hyper K-algebra since does not exist a proper subset Ω of H such that $(\Omega, \circ, 0)$ is a non-trivial hyper BCK-algebra.

Lemma 3.7. ([1, Theorem 3.9]) *Let $(H_1, \circ_1, 0)$ and $(H_2, \circ_2, 0)$ be hyper K-algebras (resp. hyper BCK-algebras) and $H = H_1 \times H_2$. We define a hyperoperation "$\circ$" on H is defined as follows,*

$$(a_1, b_1) \circ (a_2, b_2) = (a_1 \circ a_2, b_1 \circ b_2)$$

for all $(a_1, b_1), (a_2, b_2) \in H$, where for $A \subseteq H_1$ and $B \subseteq H_2$ by (A, B) we mean

$$(A, B) = \{(a, b) : a \in A, b \in B\},\ 0 = (0_1, 0_2)$$

and

$$(a_1, b_1) < (a_2, b_2) \Leftrightarrow a_1 < a_2, b_1 < b_2.$$

Then $(H, \circ, 0)$ is a hyper K-algebra (resp. hyper BCK-algebra), and it is called the hyper K-product (resp. hyper BCK-product) of H_1 and H_2.

Theorem 3.8. *Let $(H_1, \circ_1, 0)$ and $(H_2, \circ_2, 0)$ be hyper K-algebras. If $(H_1, \circ_1, 0)$ is a Smarandache hyper K-algebra or $(H_2, \circ_2, 0)$ is a Smarandache hyper K-algebra, then the hyper K-product $H = H_1 \times H_2$ of H_1 and H_2 is also a Smarandache hyper K-algebra.*

Proof. We may assume that $(H_1, \circ_1, 0)$ is a Smarandache hyper K-algebra without loss of generality. Then there exists a non-trivial hyper BCK-algebra Ω in H_1. Let $\Gamma = \Omega \times \{0_2\}$. Then Γ is a proper subset of $H = H_1 \times H_2$ and obviously $(\Gamma, \circ, 0)$ is a non-trivial hyper BCK-algebra. Hence $H = H_1 \times H_2$ is a Smarandache hyper K-algebra. □

The following example shows that the converse of Theorem 3.8 is not true in general.

Example 3.9. Let $H_1 = \{0_1, x\}$ and $H_2 = \{0_2, y\}$ and define the hyperoperations "$\circ_1$" and "$\circ_2$" on H_1 and H_2 respectively as follow:

$\circ_1$	0_1	x
0_1	$\{0_1\}$	$\{0_1\}$
x	$\{x\}$	$\{0_1, x\}$

$\circ_2$	0_2	y
0_2	$\{0_2\}$	$\{0_2\}$
y	$\{0_2, y\}$	$\{0_2\}$

Then H_1 is a hyper BCK-algebra and H_2 is a hyper K-algebra. We know that $(H_1 \times H_2, \circ, 0 = (0_1, 0_2))$ is a hyper K-algebra with the following Cayley table:

$\circ$	$(0_1, 0_2)$	$(0_1, y)$	$(x, 0_2)$	(x, y)
$(0_1, 0_2)$	$(0_1, 0_2)$	$(0_1, 0_2)$	$(0_1, 0_2)$	$(0_1, 0_2)$
$(0_1, y)$	$(0_1, \{0_2, y\})$	$(0_1, 0_2)$	$(0_1, \{0_2, y\})$	$(0_1, 0_2)$
$(x, 0_2)$	$(x, 0_2)$	$(x, 0_2)$	$(\{0_1, x\}, 0_2)$	$(\{0_1, x\}, 0_2)$
(x, y)	$(x, \{0_2, y\})$	$(x, 0_2)$	$(\{0_1, x\}, \{0_2, y\})$	$(\{0_1, x\}, 0_2)$

Now a proper subset $H_1 \times \{0\}$ of $H_1 \times H_2$ is a non-trivial hyper BCK-algebra. Thus $H_1 \times H_2$ is a Smarandache hyper K-algebra. But we know that neither H_1 nor H_2 is a Smarandache hyper K-algebra.

Proposition 3.10. *Let $(H_1, \circ_1, 0)$ and $(H_2, \circ_2, 0)$ be hyper K-algebras. If $(H_1 \times H_2, \circ, 0)$, the hyper K-product of H_1 and H_2, is a Smarandache hyper K-algebra, then at least one of H_1 and H_2 is a Smarandache hyper K-algebra.*

Proof. Let $(H_1 \times H_2, \circ, 0)$ be a Smarandache hyper K-algebra. Then there exists a proper subset Ω of $H_1 \times H_2$ such that $(\Omega, \circ, 0)$ is a non-trivial hyper BCK-algebra. Let $\Omega_1 = \{x \in H_1 : (x, b) \in \Omega,$ for some $b \in H_2\}$ and $\Omega_2 = \{y \in H_2 : (a, y) \in \Omega,$ for some $a \in H_1\}$. It is easily verified that $\Omega = \Omega_1 \cup \Omega_2$. Let $x, y, z \in \Omega_1$. Then there exist $a, b, c \in H_2$ such that $(x, a), (y, b), (z, c) \in \Omega$. Now we show that $(\Omega_1, \circ_1, 0)$ is a hyper BCK-algebra.

(HK1) Since $(\Omega, \circ, 0)$ satisfies the condition (HK1), we have

$$((x,a) \circ (z,c)) \circ ((y,b) \circ (z,c)) \ll (x,a) \circ (y,b),$$

that is,

$$((x \circ_1 z) \circ_1 (y \circ_1 z), (a \circ_2 c) \circ_2 (b \circ_2 c)) \ll (x \circ_1 y, a \circ_2 b).$$

Hence $(x \circ_1 z) \circ_1 (y \circ_1 z) \ll x \circ_1 y$ and so (HK1) holds in $(\Omega_1, \circ_1, 0)$.

(HK2) Since $(\Omega, \circ, 0)$ satisfies the condition (HK2), we have

$$((x,a) \circ (y,b)) \circ (z,c) = ((x,a) \circ (z,c)) \circ (y,b),$$

which implies that $((x \circ_1 y) \circ_1 z, (a \circ_2 b) \circ_2 c) = ((x \circ_1 z) \circ_1 y, (a \circ_2 c) \circ_2 b)$. Hence, we get $(x \circ_1 y) \circ_1 z = (x \circ_1 z) \circ_1 y$ and so (HK2) holds in $(\Omega_1, \circ_1, 0)$.

(HK3) Since $(\Omega, \circ, 0)$ satisfies the condition (HK3), we have $(x, a) \circ (y, b) \ll (x, a)$, which implies that $(x \circ_1 y, a \circ_2 b) \ll (x, a)$. Hence, we get $x \circ_1 y \ll x$ and so (HK3) holds in $(\Omega_1, \circ_1, 0)$.

(HK4) Let $(x, a) \ll (y, b)$ and $(y, b) \ll (x, a)$. Since $(\Omega, \circ, 0)$ satisfies the condition (HK4), we have $(x, a) = (y, b)$. Hence, we get $x = y$ and so (HK4) holds in $(\Omega_1, \circ_1, 0)$.

Thus, $(\Omega_1, \circ_1, 0)$ is a hyper BCK-algebra. In the similar way we can show that $(\Omega_2, \circ_2, 0)$ is a hyper BCK-algebra. It follows from $\Omega \neq (0, 0)$ that $\Omega_1 \neq 0$ or $\Omega_2 \neq 0$. Without loss of generality we may assume that $\Omega_1 \neq 0$. Note that $\Omega_1 \subseteq H_1$, but $\Omega_1 \neq H_1$ since H_1 is a proper hyper K-algebra. Hence, Ω_1 is a proper subset of H_1 such that $(\Omega_1, \circ_1, 0)$ is a non-trivial hyper BCK-algebra. Therefore H_1 is a Smarandache hyper K-algebra. □

Proposition 3.11. *Let $(H_1, \circ_1, 0)$ and $(H_2, \circ_2, 0)$ be hyper K-algebras such that $H_1 \cap H_2 = \{0\}$. If at least one of H_1 and H_2 is a Smarandache hyper K-algebra, then $(H_1 \oplus H_2, \circ, 0)$, the union of H_1 and H_2, is also a Smarandache hyper K-algebra.*

Proof. Let $(H_1, \circ_1, 0)$ and $(H_2, \circ_2, 0)$ be hyper K-algebras such that $H_1 \cap H_2 = \{0\}$. Without loss of generality we may assume H_1 is a Smarandache hyper K-algebra. Then there exists a proper subset Ω of H_1 such that $(\Omega, \circ_1, 0)$ is a non-trivial hyper BCK-algebra. Since $H_1 \subseteq H_1 \oplus H_2$, Ω is a proper subset of $H_1 \oplus H_2$. By the definition of hyperoperation "$\circ$" on $H_1 \oplus H_2$ and $\Omega \subseteq H_1$, we have $(\Omega, \circ_1, 0) = (\Omega, \circ, 0)$. Hence, Ω is a proper subset of $H_1 \oplus H_2$ such that $(\Omega, \circ, 0)$ is non-trivial hyper BCK-algebra and so $H_1 \oplus H_2$ is a Smarandache hyper K-algebra. □

The following example shows that the converse of Proposition 3.11 may not be true.

Example 3.12. Consider the hyper K-algebras $H_1 = \{0, x\}$ and $H_2 = \{0, y\}$ as in Example 3.9, where $0 = 0_1 = 0_2$. It is easily verified that $(H_1 \oplus H_2, \circ, 0)$ is a hyper K-algebra under

the following Cayley table.

$\circ$	0	x	y
0	$\{0\}$	$\{0\}$	$\{0\}$
x	$\{x\}$	$\{0,x\}$	$\{x\}$
y	$\{0,y\}$	$\{y\}$	$\{0\}$

Using the above table it is easily verified that $(\{0,x\},\circ,0)$ is a hyper BCK-algebra. Therefore, $H_1 \oplus H_2$ is a Smarandache hyper K-algebra. But H_1 and H_2 are not Smarandache hyper K-algebra, since $|H_1| = 2 = |H_2|$.

Proposition 3.13. *Let $(H_1, \circ_1, 0)$ and $(H_2, \circ_2, 0)$ be hyper K-algebras such that $H_1 \cap H_2 = \{0\}$. If $(H_1 \oplus H_2, \circ, 0)$ is a Smarandache hyper K-algebra, then at least one of H_1 and H_2 is a Smarandache hyper K-algebra.*

Proof. Let $(H_1 \oplus H_2, \circ, 0)$ be a Smarandache hyper K-algebra. Then there exists a proper subset Ω of $H_1 \oplus H_2$ such that $(\Omega, \circ, 0)$ is a non-trivial hyper BCK-algebra. Assume that $\Omega_1 = \Omega \cap H_1$ and $\Omega_2 = \Omega \cap H_2$. Then $\Omega = \Omega_1 \cup \Omega_2$, and so $\Omega_1 \neq \{0\}$ or $\Omega_2 \neq \{0\}$. Without loss of generality we may assume that $\Omega_1 \neq \{0\}$. Since $x \circ y = x \circ_1 y$ for all $x, y \in \Omega_1$, we have $(\Omega_1, \circ_1, 0) = (\Omega_1, \circ, 0)$. Let $x, y \in H_1$ and $x, y \in \Omega$. Then $x \circ y = x \circ_1 y \in H_1$ and $x \circ y \in \Omega$. Therefore $x \circ y \in \Omega_1$. This shows that Ω_1 is a hyper subalgebra of Ω. Hence, $(\Omega_1, \circ, 0) = (\Omega_1, \circ_1, 0)$ is a non-trivial hyper BCK-algebra. Obviously Ω_1 is a proper subset of H_1. Therefore H_1 is a Smarandache hyper K-algebra. □

Definition 3.14. Let H be a Smarandache hyper hyper K-algebra, Ω be a non-trivial hyper BCK-algebra which is properly contained in H. Then a non-empty subset I of H is called a *Smarandache hyper $(\cap, \in)$-ideal* of H related to Ω (or briefly, *Ω-Smarandache hyper $(\cap, \in)$-ideal*) of H if it satisfies:

(c1) $0 \in I$,

(cs) $(\forall x \in \Omega)(\forall y \in I)((x \circ y) \cap I \neq \emptyset \Rightarrow x \in I)$.

If I is a Smarandache hyper $(\cap, \in)$-ideal of H related to every hyper BCK-algebra contained in H, we simply say that I is a *Smarandache hyper $(\cap, \in)$-ideal* of H.

Example 3.15. Let $H = \{0, a, b, c\}$ and define the hyperoperation "$\circ$" on H by the following Cayley table:

$\circ$	0	a	b	c
0	$\{0\}$	$\{0\}$	$\{0\}$	$\{0\}$
a	$\{a\}$	$\{0\}$	$\{a\}$	$\{a\}$
b	$\{b\}$	$\{b\}$	$\{0,b\}$	$\{0,b\}$
c	$\{c\}$	$\{c\}$	$\{b,c\}$	$\{0,b,c\}$

Then $(H, \circ, 0)$ is a Smarandache hyper K-algebra because $(\Omega = \{0, a, b\}, \circ, 0)$ is a hyper BCK-algebra. Moreover, a subset $\{0, a\}$ is an Ω-Smarandache hyper $(\cap, \in)$-ideal of H.

Theorem 3.16. *Let H be a Smarandache hyper hyper K-algebra, Ω be a non-trivial hyper BCK-algebra which is properly contained in H. Then every Ω-Smarandache hyper $(\cap, \in)$-ideal of H is an Ω-Smarandache hyper $(\ll, \in)$-ideal of H.*

Proof. Let I be an Ω-Smarandache hyper $(\cap, \in)$-ideal of H and let $x \in \Omega$ and $y \in I$ be such that $x \circ y \ll I$. Then for any $a \in x \circ y$ there exists $i \in I$ such that $a \ll i$, which implies that $0 \in a \circ i$. Hence $(a \circ i) \cap I \neq \emptyset$ and so by (cs), we have $a \in I$. This implies that $(x \circ y) \cap I \neq \emptyset$ and so by (cs) we have $x \in I$. □

The following example shows that the converse of Theorem 3.16 may not be true.

Example 3.17. Let $H = \{0, a, b, c\}$ and define the hyperoperation "$\circ$" on H by the following Cayley table:

$\circ$	0	a	b	c
0	$\{0\}$	$\{0\}$	$\{0\}$	$\{0\}$
a	$\{a\}$	$\{0, a\}$	$\{0, a\}$	$\{a\}$
b	$\{b\}$	$\{a, b\}$	$\{0, a, b\}$	$\{0, b\}$
c	$\{c\}$	$\{c\}$	$\{b, c\}$	$\{0, a, b, c\}$

Then $(H, \circ, 0)$ is a Smarandache hyper K-algebra because $(\Omega = \{0, a, b\}, \circ, 0)$ is a hyper BCK-algebra. Moreover, a subset $I = \{0, a\}$ is an Ω-Smarandache hyper $(\ll, \in)$-ideal of H. But it is not an Ω-Smarandache hyper $(\cap, \in)$-ideal of H, since $(b \circ a) \cap I \neq \emptyset$ and $a \in I$, but $b \notin I$.

Corollary 3.18. *Let H be a Smarandache hyper hyper K-algebra, Ω be a non-trivial hyper BCK-algebra which is properly contained in H. Then every Ω-Smarandache hyper $(\cap, \in)$-ideal of H is an Ω-Smarandache hyper $(\subseteq, \in)$-ideal of H.*

Proof. The result is obvious by Theorem 3.16 and Theorem 3.16 in [3]. $\square$

Theorem 3.19. *Let H be a Smarandache hyper hyper K-algebra, Ω be a non-trivial hyper BCK-algebra which is properly contained in H and let I be an Ω-Smarandache hyper $(\ll, \in)$-ideal of H such that*

$$(\forall x \in \Omega)(x \circ x \subseteq I \subseteq \Omega).$$

Then the following implication is valid:

$$(\forall x, y \in \Omega)((x \circ y) \cap I \neq \emptyset \Rightarrow x \circ y \subseteq I).$$

Proof. Let $x, y \in \Omega$ be such that $(x \circ y) \cap I \neq \emptyset$. Then there exists $t \in \Omega$ such that $t \in (x \circ y) \cap I$. It follows from (HK1) that $(x \circ y) \circ (x \circ y) \ll x \circ x$ so from hypothesis that $(x \circ y) \circ (x \circ y) \ll I$. This implies that $s \circ t \ll I$ for all $s \in x \circ y$, and hence $s \in I$ since I is an Ω-Smarandache hyper $(\ll, \in)$-ideal of H and $t \in I$. Therefore $x \circ y \subseteq I$. $\square$

Theorem 3.20. *Let H be a Smarandache hyper hyper K-algebra, Ω be a non-trivial hyper BCK-algebra which is properly contained in H and let I be an Ω-Smarandache hyper $(\ll, \in)$-ideal of H such that*

$$(\forall x \in \Omega)(x \circ x \subseteq I \subseteq \Omega).$$

Then I be an Ω-Smarandache hyper $(\cap, \in)$-ideal of H.

Proof. Let $x, y \in \Omega$ be such that $(x \circ y) \cap I \neq \emptyset$ and $y \in I$. Then $x \circ y \subseteq I$ by Theorem 3.19, and so $x \circ y \ll I$. Since I is an Ω-Smarandache hyper $(\ll, \in)$-ideal of H, it follows that $x \in I$. Therefore I is an Ω-Smarandache hyper $(\cap, \in)$-ideal of H. $\square$

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